# 3.2 – Polynomial Functions

so far, we've looked at

lines
parabolas,
exponentials,
& logarithms

these are the nicest, & the most common

We definitely understand lines & parabolas the best.

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OO FAST

We will come back to exponentials & logs after exam II,

But we'll start by looking at something a bit easier.

lines (degree 1) and parabolas (degree 2) are both polynomials.

Define: if $n$ is a positive integer,
and if $a_n, a_{n-1}, \dots, a_2, a_1, a_0$ are #'s

$$f(x) = a_n \cdot x^{n} + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

degree

variable

is a polynomial

in $x$

of degree $n$

$\boxed{\text{degree} = \text{highest power of } x}$

## The anatomy of a polynomial

$f(x) =$ [redacted] $4x^3$ [redacted] $+ 2x + 1$

[redacted]

**degree**: 3

**leading coefficient**: 4 [redacted]

**leading term**: [redacted] $4x^3$

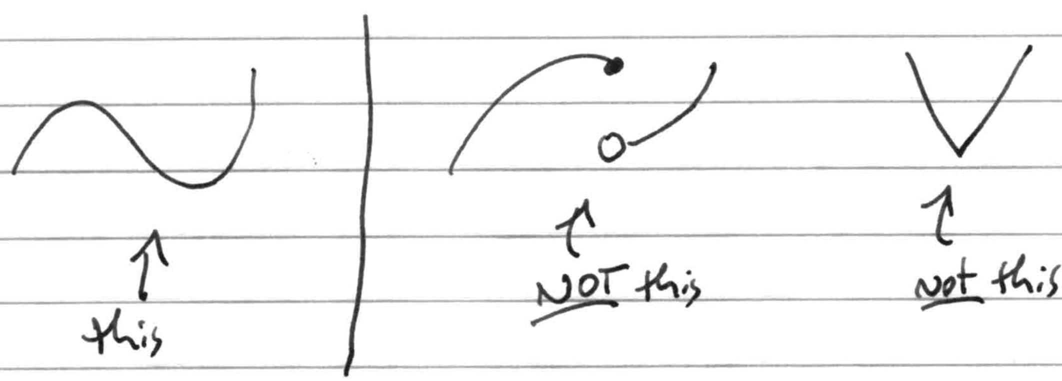
**constant term**: 1

VERY EASY
5 [redacted]

Our main goal is [redacted]
learning to sketch
a polynomial $p(x)$

**Goal**: [redacted] Sketching Polynomials

**Fact 1**: polynomials are continuous,
& have no sharp corners



<u>Fact 2:</u> when $x$ is big (positive or negative)

$$f(x) = 4x^3 + 2x + 1 \approx 4x^3$$

> In words:
> the "end behavior" of $f(x)$ is described by its leading term.

Graphing the leading term is easy:

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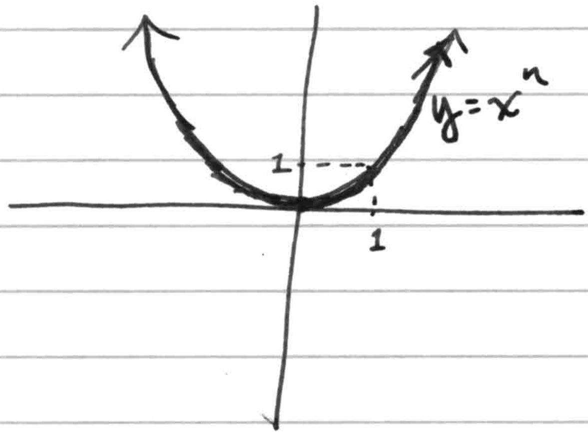
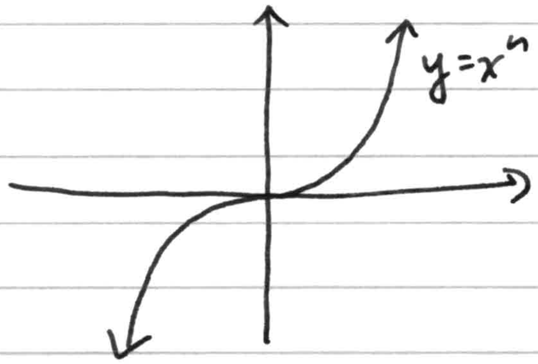
If $n$ is <u>even</u>,




the bigger $n$ is
the tighter the corners are

Read the book for <u>helpful</u> pictures

If $n$ is <u>odd</u>

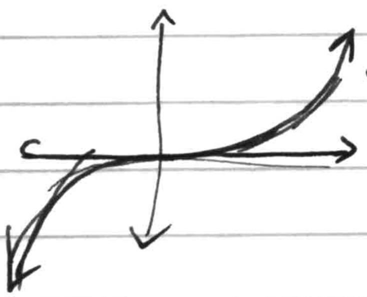



the bigger $n$ is
the tighter the corners are

Eg: The leading term of

$$f(x) = x^3 + 4x + 10$$

is $x^3$



← the graph of the leading term

VERY FAST

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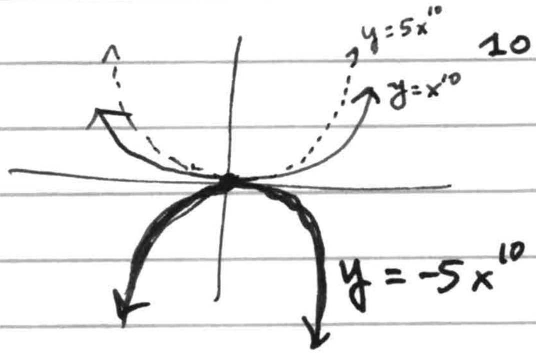
Eg: the leading term of

$$f(x) = -5x^{10} + 4x^7 + 5x$$

is $-5x^{10}$

10 is even



## To Sketch a degree n polynomial $f(x)$

you just need

step 1: a sketch of the leading term

$$y = x^n$$

(this [redacted] shows "eventual" behavior)

step 2: where is $f(x) = 0$ ?

5 (these are the x-intercepts)

step 3: when is [redacted] $f(x)$ positive/negative?

(this lets you fill in the "middle" behavior)

SKIP IN CLASS

Eg: Sketch a polynomial $h(x)$

with ① leading coefficient $x^3$

② zero's at $-1$, $0$, and $1$

③ that is positive on $(-1,0) \cup (1,\infty)$

and negative on $(-\infty,-1) \cup (0,1)$

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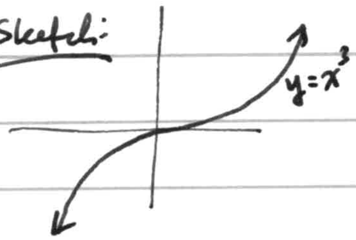
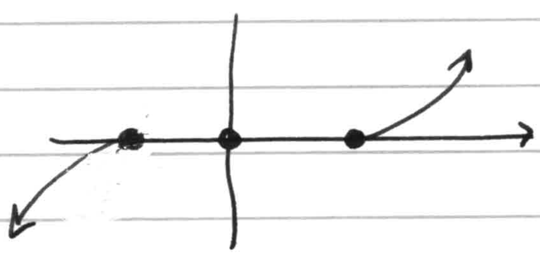
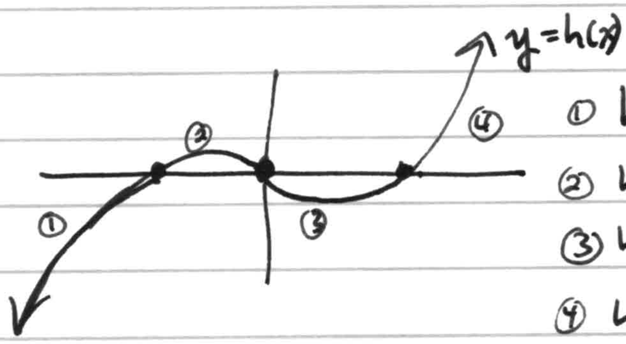
Sketch:




when $x$ is big
the leading coefficient wins
so this is $h(x)$ zoomed far out



$h(x)$ hits the $x$ axis at
$-1$, $0$, & $1$




① $h(x) < 0$ left of $1$

② $h(x) > 0$ on $(-1,0)$

③ $h(x) < 0$ on $(0,1)$

④ $h(x) > 0$ on $(1,\infty$

⇑

$\boxed{\text{this is our sketch}}$

**Eg:** Sketch the graph of

$$h(x) = x^3 - x$$

① leading term is $x^3$

$\Rightarrow$ end-behavior is 

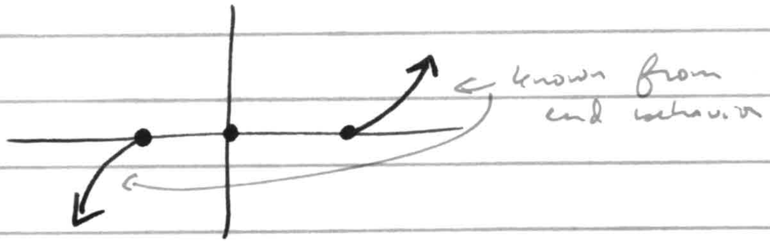
② $h(x) = 0$

$\Rightarrow \quad x^3 - x = 0$

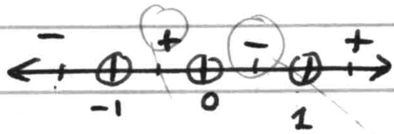
10-15 $\quad x(x^2-1) = 0$

$x(x+1)(x-1) = 0$

$\Rightarrow$ zero when $\quad x=0$ or $x=-1$ or $x=1$




③ $h(x) > 0$




$\Rightarrow \quad x^3 - x > 0$

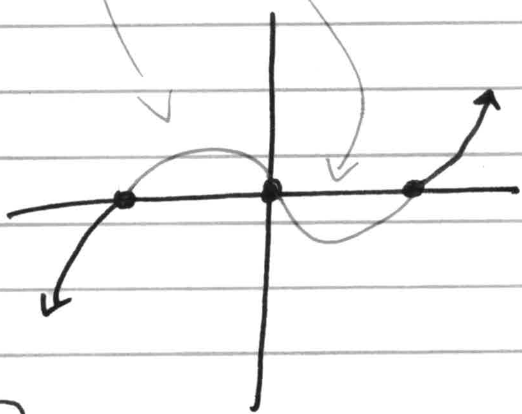
$\Rightarrow \quad x(x+1)(x-1) > 0$

$x=-2 \quad \Rightarrow \quad (\text{neg})(\text{neg})(\text{neg}) = (\text{neg})$

$x=-\frac{1}{2} \quad \Rightarrow \quad (\text{neg})(\text{pos})(\text{neg}) = \text{pos}$

$x=\frac{1}{2} \quad \Rightarrow \quad (\text{pos})(\text{pos})(\text{neg}) = \text{neg}$

$x=2 \quad \Rightarrow \quad (\text{pos})(\text{pos})(\text{pos}) = \text{pos}$



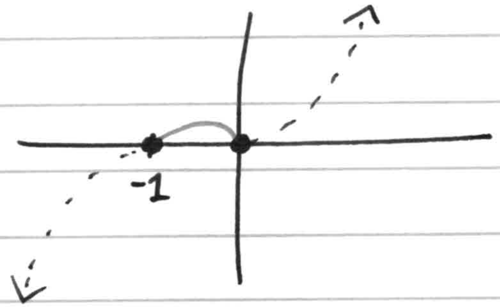
See textbook Eg's 6, 7, 8 for more

Something interesting [redacted] happens

if [redacted] a factor appears multiple times

Eg: graphing $h(x) = x^3 + x^2 = x^2(x+1)$

gives




If the factor $(x-c)$ occurs

I. an even # of times,
the graph <u>doesn't</u> cross the x-axis at $(c,0)$

II. an odd # of times,
the graph <u>does</u> cross the x-axis

The number of times $(x-c)$ goes into $h(x)$ is called the <u>multiplicity of the zero at $c$</u>.